

Solving for Oxygen Exchange Rate

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1 Purpose and Derivation

Given a device which is filled with 1 atm of nitrogen and leaking into ambient air, we want to solve for the time required for the oxygen in air to reach a specified concentration within the device. We start with the molecular flow equation from Howl and Mann.¹

$$VP'(t) = \frac{K}{\sqrt{M}}(P_E - P(t)) \quad (1)$$

Integrate to solve for $P(t)$, and set $P(0) = P_o = 0$ as the initial condition, as there is no oxygen initially present inside the device.

$$P(t) = P_E \left(1 - e^{\frac{-Kt}{V\sqrt{M}}} \right) \quad (2)$$

Make the substitution $K = L_{He}\sqrt{M_{He}}$. This allows use of equation in terms of helium leak rate.

$$P(t) = P_E \left(1 - e^{\frac{-L_{He}\sqrt{M_{He}}t}{V\sqrt{M}}} \right) \quad (3)$$

Let $\frac{P(t)}{P_E} = x$, where x is the percentage of atmospheric oxygen that has leaked into the device. i.e. $P_E = .2095$. Then, solve for time.

$$t = \frac{-V\sqrt{M}}{L_{He}\sqrt{M_{He}}} \ln(1 - x) \quad (4)$$

¹C A Mann D A Howl. "The Back-Pressurising Technique of Leak-Testing". In: *Vacuum* 15(7) (1965), pp. 347-352.

2 Variables

Symbol	Description
t	Time (sec)
V	Internal free volume of device (cc)
$P(t)$	Internal partial pressure of O ₂ (atm)
P_E	External partial pressure of O ₂ (atm)
x	Percent of atmospheric O ₂ pressure reached within device.
M	Molecular weight of O ₂ (g/mol)
M_{He}	Molecular weight of O ₂ (g/mol)
K	Constant for unknown geometry of leak

References

- D A Howl, C A Mann. "The Back-Pressurising Technique of Leak-Testing". In: *Vacuum* 15(7) (1965), pp. 347–352.